İZMİR INSTITUTE OF TECHNOLOGY
GRADUATE SCHOOL OF ENGINEERING AND SCIENCES
DEPARTMENT OF MATHEMATICS
CURRICULUM OF THE GRADUATE PROGRAMS

M.S. in Mathematics (Thesis)

Core Courses

MATH 596	Graduate Seminar	(0-2) NC AKTS:9
MATH 599	Scientific Research Techniques and Publication Ethics	(0-2) NC AKTS:8
MATH 500	M.S. Thesis	(0-1) NC AKTS:26
MATH 8XX	Special Studies	(8-0) NC AKTS:4

In addition, the following courses must be taken.

MATH 515	Real Analysis	(3-0)3 AKTS:8
MATH 527	Basic Abstract Algebra	(3-0)3 AKTS:8

*All M.S. students must register Graduate Seminar course until the beginning of their 4^{th} semester.

Total credit (min.) :21
Number of courses with credit (min.) : 7

M.S. in Mathematics (Non-thesis)

Core Courses

MATH 516	Complex Analysis	(3-0)3 AKTS:8
MATH 527	Basic Abstract Algebra	(3-0)3 AKTS:8
MATH 533	Ordinary Differential Equations	(3-0)3 AKTS:8
MATH 534	Partial Differential Equations	(3-0)3 AKTS:8
MATH 573	Modern Geometry I	(3-0)3 AKTS:8
MATH 595	Graduation Project	(0-2) NC AKTS:5
MATH 596	Graduate Seminar	(0-2) NC AKTS:9
MATH 599	Scientific Research Techniques and Publication Ethics	(0-2) NC AKTS:8

Total credit (min.) :30
Number of courses with credit (min.) :10

Ph.D. in Mathematics

Core Courses

MATH 597	Comprehensive Studies	(0-2) NC AKTS:9
MATH 598	Graduate Seminar in PhD	(0-2) NC AKTS:9
MATH 599	Scientific Research Techniques and Publication Ethics*	(0-2) NC AKTS:8
MATH 600	Ph.D. Thesis	(0-1) NC AKTS:26
MATH 8XX	Special Studies	(8-0) NC AKTS:4

*This course is not obligatory for the PhD students who already pass this course during their M.S. study.

Total credit (min.) :21 (for students with M.S. degree)
Number of courses with credit (min.): 7 (for students with M.S. degree)

Total credit (min.) : 42 (for students with B.S. degree)
Number of courses with credit (min.): 14 (for students with B.S. degree)

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Elective Courses

MATH 501	Combinatorial Design Theory	(3-0)3 ECTS:7
MATH 502	**Harmonic Analysis**	**(3-0)3 ECTS:7**
MATH 503	Inverse Problems	(3-0)3 ECTS:7
MATH 505	Fundamental Methods in Discrete Mathematics	(3-0)3 ECTS:7
MATH 506	Combinatorics	(3-0)3 ECTS:7
MATH 507	**Stochastic Calculus and Finance**	**(3-0)3 ECTS:7**
MATH 508	Graph Theory	(3-0)3 ECTS:7
MATH 510	Hilbert Space Theory with Applications	(3-0)3 ECTS:7
MATH 511	Algorithms I	(3-0)3 ECTS:7
MATH 512	Algorithms II	(3-0)3 ECTS:7
MATH 513	Mathematical Methods of Fluid Mechanics	(3-0)3 ECTS:7
MATH 515	Real Analysis	(3-0)3 ECTS:8
MATH 516	Complex Analysis	(3-0)3 ECTS:8
MATH 517	Advanced Linear Algebra	(3-0)3 ECTS:8
MATH 518	Numerical Linear Algebra	(3-0)3 ECTS:7
MATH 519	Methods of Mathematical Physics	(3-0)3 ECTS:8
MATH 520	**Mathematical Models of Nonlinear Waves**	**(3-0)3 ECTS:7**
MATH 521	Module and Ring Theory I	(3-0)3 ECTS:7
MATH 522	Module and Ring Theory II	(3-0)3 ECTS:7
MATH 523	Algebraic Topology I	(3-0)3 ECTS:7
MATH 524	Algebraic Topology II	(3-0)3 ECTS:7
MATH 525	Introduction to Homological Algebra	(3-0)3 ECTS:8
MATH 526	Relative Homological Algebra	(3-0)3 ECTS:7
MATH 527	Basic Abstract Algebra	(3-0)3 ECTS:8
MATH 528	Set Theory	(3-0)3 ECTS:7
MATH 529	Abelian Groups	(3-0)3 ECTS:7
MATH 530	Quantum Calculus	(3-0)3 ECTS:7
MATH 531	Numerical Solution of ODE	(3-0)3 ECTS:8
MATH 533	Ordinary Differential Equations	(3-0)3 ECTS:8
MATH 534	Partial Differential Equations	(3-0)3 ECTS:8
MATH 535	Perturbation Method for Differential Equations	(3-0)3 ECTS:7
MATH 536	Geometric Integration	(3-0)3 ECTS:7
MATH 539	Numerical Analysis	(3-0)3 ECTS:8
MATH 540	Numerical Solution of Partial Differential Equations	(3-0)3 ECTS:7
MATH 541	Ring Theory	(3-0)3 ECTS:8
MATH 542	Algebraic Number Theory	(3-0)3 ECTS:8
MATH 543	Commutative Algebra I	(3-0)3 ECTS:7
MATH 544	Commutative Algebra II	(3-0)3 ECTS:7
MATH 546	Advanced Module Theory	(3-0)3 ECTS:7
MATH 547	**Selected Topics in Rings and Modules**	**(3-0)3 ECTS:7**
MATH 548	Introduction to Algebraic Geometry	(3-0)3 ECTS:7
MATH 549	Selected Topics in Algebraic Geometry	(3-0)3 ECTS:7
MATH 554	Brownian Motion and Schrödinger's Equation	(3-0)3 ECTS:7
MATH 562	**Applied Functional Analysis**	**(3-0)3 ECTS:7**
MATH 563	Introduction to Finite Elements	(3-0)3 ECTS:7
MATH 564	Functional Analysis	(3-0)3 ECTS:8
MATH 565	Introduction to Spectral Theory	(3-0)3 ECTS:7

MATH 566	Mathematical Foundations of Finite Element Method	(3-0)3 ECTS:7
MATH 567	Mathematical Methods of Quantum Mechanics I	(3-0)3 ECTS:7
MATH 568	Mathematical Methods of Quantum Mechanics II	(3-0)3 ECTS:7
MATH 569	Basic Quantum Computation and Quantum Information	(3-0)3 ECTS:7
MATH 570	**Theory of Bounded Operators**	**(3-0)3 ECTS:7**
MATH 571	Mathematical Methods of Classical Mechanics I	(3-0)3 ECTS:7
MATH 572	Mathematical Methods of Classical Mechanics II	(3-0)3 ECTS:7
MATH 573	Modern Geometry I	(3-0)3 ECTS:8
MATH 574	Modern Geometry II	(3-0)3 ECTS:7
MATH 575	Integral Equations	(3-0)3 ECTS:7
MATH 576	Introduction to Solution Theory	(3-0)3 ECTS:7
MATH 577	Supersymmetric Quantum Mechanics	(3-0)3 ECTS:7
MATH 578	**Asymptotic Analysis**	**(3-0)3 ECTS:7**
MATH 579	**Applied Complex Analysis**	**(3-0)3 ECTS:7**
MATH 581	Topology I	(3-0)3 ECTS:7
MATH 582	Topology II	(3-0)3 ECTS:7 **Pre. MATH 581**
MATH 584	Partial Differential Equations II	(3-0)3 ECTS:8
MATH 585	Symmetries and Groups	(3-0)3 ECTS:7
MATH 586	Hilbert Spaces and Quantum Theory	(3-0)3 ECTS:7
MATH 587	Mathematics of Public Key Cryptography	(3-0)3 ECTS:7
MATH 588	**Mathematical Control Theory for Infinite Dimensional Systems**	**(3-0)3 ECTS:7**
MATH 589	Advanced Combinatorics	(3-0)3 ECTS:7
MATH 590	Selected Topics in Applied Mathematics	(3-0)3 ECTS:7 **Pre. Consent of Instructor**
MATH 595	Graduation Project	(0-2) Non-Credit ECTS:5
MATH 596	Graduate Seminar	(0-2) Non-Credit ECTS:9
MATH 597	Comprehensive Studies	(0-2)Non-Credit ECTS:9
MATH 598	Graduate Seminar in PhD	(0-2) Non-Credit AKTS:9
MATH 599	Scientific Research Techniques and Publication Ethics	(0-2) Non-Credit AKTS:8
MATH 500	M.S. Thesis	(0-1) Non-Credit ECTS:26
MATH 600	Ph.D. Thesis	(0-1) Non-Credit ECTS:26
MATH 8XX	Special Studies	(8-0) Non-Credit ECTS:4

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COURSE DESCRIPTIONS

MATH 501 Combinatorial Design Theory (3-0)3 ECTS 7
An overview of combinatorial design theory, main constructions and theorems. Relations with finite affine and projective spaces, as well as error correcting codes.

MATH 502 Harmonic Analysis (3-0)3 ECTS 7
Starting with the foundational work of J. Fourier on solving differential equations using trigonometric series, we will cover the Fourier series, the Fourier transform, and their algebraic and spectral theoretic properties and generalizations, which highlight their essential unity. We then will concentrate on the Euclidean spaces and introduce oscillatory integrals, singular integrals and maximal operators, which altogether form the three main branches of study in modern Euclidean harmonic analysis, and which are deeply interrelated. Oscillatory integrals generalize the Fourier transform, singular integrals generalize the Hilbert transform and are related to the Cauchy integrals of complex analysis, and maximal operators generalize the Hardy-Littlewood maximal function and are extremely relevant to proving existence of limits. We then deepen our investigation of oscillatory integrals by investigating multipliers. We finally turn our attention to analyzing functions by dividing them into building blocks other than characters. This subject is known as time-frequency analysis and is of profound importance in engineering as it allows better localized and less structured building blocks.

MATH 503 Inverse Problems (3-0)3 ECTS 7
Well-posed and ill-posed problems in the sense of Hadamard, regularization methods, inverse problems, Radon transform, Shannon sampling theorem, Algebraic Reconstruction Technique (ART).

MATH 505 Fundamental Methods in Discrete Mathematics (3-0)3 ECTS 7
Counting methods and techniques; generating functions; formal power series; binomial theorem; recurrence relations and their solutions; graph terminology; adacency and incidence matrices; isomorphism; matchings; planar graphs; chromatic number; stable sets and cliques; connectivity; growth of functions; running times; complexity classes.

MATH 506 Combinatorics (3-0)3 ECTS 7
Topics in strings on finite alphabets; topics in integer partitions; topics in symmetric functions; topics in error correcting codes; topics in combinatorial designs; topics in lattices and posets; topics in representation.

MATH 507 Stochastic Calculus and Finance (3-0)3 ECTS 7
The course will start with an overview of finance, importance of efficient financing and securities as the means of this. Afterwards the fundamentals of probability theory will be covered, Brownian motion as the main tool of modeling asset prices will be introduced and developed. Combining this model and probability theory with basic principles of asset pricing we will deduce price formulas for different types of securities. We then will develop the machinery of calculus for Brownian motion paths, and apply it to asset pricing.

MATH 508 Graph Theory (3-0)3 ECTS 7
Connectivity and Menger's theorem; embeddings of graphs and Kuratowski's theorem; network flows; crossing number; structure of k-chromatic graphs; Hamiltonian cycles; decomposition and packing of graphs.

MATH 510 Hilbert Space Theory with Applications (3-0)3 ECTS 7
The Lebesque integral. Hilbert spaces. Linear operators. Fredholm integral equation. Voltera integral equation, Applications to ordinary differential equations. Sturm-Liouville systems. Inverse differential

operators and Green’s functions. Application of Fourier transform to ordinary differential and integral equations. Generalized functions. Fundamental solutions and Green’s function for partial differential equations. Week solutions of elliptic boundary value problems, Applications of Fourier transform to partial differential equations. Miscellaneous applications to equations of mathematical physics.

MATH 511 Algorithms I **(3-0)3 ECTS 7**
Combinatorial optimization, integer programming, network and graph algorithms, algorithms in cryptography and cryptographic protocols, group theoretic algorithms, geometric algorithms.

MATH 512 Algorithms II **(3-0)3 ECTS 7**
Random events; probabilistic methods; randomized algorithms; approximation algorithms.

MATH 513 Mathematical Methods of Fluid Mechanics **(3-0)3 ECTS 7**
Euler’s Equations, rotation and vorticity. The Navier- Stokes equations. Potential flow. Boundary layers. Vortex sheets. Stability and bifurcation. Characteristics. Shocks. The Riemann problem. Combustion waves.

MATH 515 Real Analysis **(3-0)3 ECTS 8**
Lebesgue measure and Lebesgue integration. The Lebesgue spaces. General measure and integration. Decomposition of measures. Radon-Nikodym theorem. Extension of measures. Product measures and Fubini's theorem.

MATH 516 Complex Analysis **(3-0)3 ECTS 8**
Analytic functions. Cauchy-Riemann equations. Harmonic functions. Elementary functions: the exponential function, trigonometric functions, hyperbolic functions. The logarithmic function and its branches. Contour Integrals and Cauchy’s theorem. Cauchy integral formula. Liouville’s theorem and the fundamental theorem of algebra. Maximum moduli of functions. Incompressible and irrotational flow. Complex potential. Laurent’s series and classification of singularities. Sources and vortices as singular points of potential flow. Calculus of residues. Conformal mappings. Fractional linear transformations. Applications of conformal mappings. Laplace’s equation. Electrostatic potential. Elements of elliptic functions. Analytic continuation and elementary Riemann surfaces.

MATH 517 Advanced Linear Algebra **(3-0)3 ECTS 8**
Vector spaces, matrices, linear mapping, scalar products and orthogonality, determinants, symmetric, hermition and unitary operators, eigenvectors and eigenvalues, polynomials and matrices, primary decomposition and jordan normal form.

MATH 518 Numerical Linear Algebra **(3-0)3 ECTS 7**
Solution of linear equations, eigenvector and eigenvalue calculation, matrix error analysis, reduction by orthogonal transformation, iterative methods.

MATH 519 Methods of Mathematical Physics **(3-0)3 ECTS 8**
Vector and tensor analysis. Potential theory and Dirac delta function matrices and groups. Continuous groups, distributions, Hilbert spaces. Differential equations, nonhomogeneous equations. The special functions I. The special functions II. Fourier series and integral transform. Laplace, Mellin and Hankel transforms. Calculus of variations. Integral equations.

MATH 520 Mathematical Models of Nonlinear Waves **(3-0)3 ECTS 8**
Linear hyperbolic equations and Green's functions. Linear dispersive waves and grup velocity. First order nonlinear partial differential equations; discontinuous solutions and shock waves. Linear and nonlinear diffusion equations. Burgers equation and Cole-Hopf transform. Solitary waves and solitons. KdV equation, nonlinear Schrödinger equation, Klein-Gordon and Sine-Gordon equations. Solution methods.

MATH 521 Module and Ring Theory I **(3-0)3 ECTS 7**
Categories of modules. Products. Coproducts. Generators and cogenerators. Injective and projective modules. Some important extensions. Injective wraps. Small epimorfizms. Projective cover. Semi simple modules and rings. Radicals and Socles of modules and rings. Radical of endomorphism ring. Semi simple-comodules and rings.

MATH 522 Module and Ring Theory II **(3-0)3 ECTS 7**
Finite modules. Coherent modules and rings. Noether modules and rings. Finite simultaneously produced modules. Artin and identical-Noether modules. Smooth modules. Uniform modules and rings. Semi-hereditary modules and rings. Completable modules. Semi-perfect modules and rings.

MATH 523 Algebraic Topology I **(3-0)3 ECTS 7**
Topologic spaces. Division axioms. Continuous functions. Path connected spaces. Compact spaces. Homotopy. Fundamental groups of topologic space. Homotopy groups. Complete sequences for homotopy groups.

MATH 525 Introduction to Homological Algebra **(3-0)3 ECTS 8**
Modules. Isomorphism theorems. Category and functor. Exact séquences. 5-Lemma.3x3-Lemma. Pullback and pushout diagrams. Functor Hom. Injective modules. Projective modules. Tensor Product. Flat modules. Relation between Hom and Tensor Product. Complexes and Homology. Injective and projective resolutions. Derived functors. Exact sequences for derived functors.

MATH 524 Algebraic Topology II **(3-0)3 ECTS 7**
Standard simplexes. Complexes of simplexes. Boundaries. Singular simplexes. Chain complexes. Singular homology group of topologic space. Homology groups of couples. Complete sequence of couple. Computation of homology group. Relations between homotopy and homology groups.

MATH 526 Relative Homological Algebra **(3-0)3 ECTS 7**
Projective and injective dimensions of modules. Global dimension of the ring. Functors Extn without projective and injective resolutions. Proper classes of short exact sequences. Examples. Operations over proper classes. Functors Extn with respect to a proper class. Projective and injective objects with respect to a proper class. Projectively and inlectively generated proper classes. Coprojective and coinjective objects. Coprojectively and coinlectively generated proper classes. Projective, injective and global dimensions with respect to proper classes.

MATH 527 Basic Abstract Algebra **(3-0)3 ECTS 8**
Integers. Sets. Linear Algebra. Groups. Subgroups. Factor groups. Isomorphism theorems. Finitely generated Abelian groups. Rings. Ideals. Maximal, prime ideals. PID. Irreducible polynomials. Fields. Algebraic extensions. Modules. Exact sequences.

MATH 528 Set Theory **(3-0)3 ECTS 7**
Partially ordered sets. Equivalence relations. Well-ordered sets. Elementary axioms of set theory: The axiom of extensionality, pairing axiom, the axiom of infinity, power set axiom, xiom of replacement. Transitive sets. Regularity. ordinal numbers. Axiom of choice. Four forms of axiom of choice. Foundation of mathematics. Cardinality. Ordinal arithmetic. Cardinal arithmetic. Cofinality.

MATH 529 Abelian Groups **(3-0)3 ECTS 7**
Abelian Groups. Direct sum and direct product. Free and divisible groups. Direct summands. Pullback and pushout diagrams. Direct and inverse limits. Topological groups. Completeness. Pure subgroups. Basic subgroups.

MATH 530 Quantum Calculus **(3-0)3 ECTS 7**
q-Drivative and h-Derivative. Generalized Taylor’s formula for polynomials Gauss’s binomial formula. q-Binomial coefficients and linear algebra over finite fields. Two Euler’s identities. Jacobi’s triple product identity. q-Hypergeometric functions. Ramanujan product formula. q-Antiderivative, q-Gamma

and q-Beta functions. Bernoulli polynomials and Bernoulli numbers. Applications in Number theory and combinatorial analysis. Applications in physics. Applications in statistics and engineering. Non-linear resonance theory of particles.

MATH 531 Numerical Solution of ODE (3-0)3 ECTS 8
Initial-value problems: Runge-Kutta, extrapolation and multistep methods. Stable methods for stiff problems. Boundary-value problems: Shooting and multiple shooting. Difference schemes, collocation. Analysis. Conditioning of boundary value problems. Consistency, stability and convergence for both initial and boundary value problems. Fourier transform tecniques. Fourier analysis, Fourier spectral methods. Geometric integrators. Lie group methods, symplectic methods, Magnus series method.

MATH 533 Ordinary Differential Equations (3-0)3 ECTS 8
This course develops techniques for solving ordinary differential equations. Topics covered include: introduction to First-Order Linear Differential Equations; Second-Order Differential Equations, existence and uniqueness theory for first order equations, power series solutions, nonlinear systems of equations and stability theory, perturbation methods, asymptotic analysis, confluent hyper geometric functions. Mathieu functions. Hill's equation.

MATH 534 Partial Differential Equations (3-0)3 ECTS 8
General theory of partial differential equations; first order equations; classification of second order equations; theory and methods of solution of elliptic, parabolic, and hyperbolic types of equations; maximum principles; Green's functions; potential theory; and miscellaneous special topics.

MATH 535 Perturbation Methods For Differential Equations (3-0)3 ECTS 7
Dimensional analysis, scaling argument, asymptotic series, Regular and singular perturbation methods for algebraic Equation and linear ordinary differential equation, nonlinear oscillation and two timing, WKP method, Laplace`s method, Stationary phases, steepest descent, boundary layer theory.

MATH 536 Geometric Integration (3-0)3 ECTS 7
Introduction to geometric integration and numerical examples, Hamiltonian mechanics and examples, geometric and non geometric integrators, symplectic maps, symplectic numerical methods, symmetric methods and reversibility, splitting and composition methods, Lie group methods, Backward error analysis.

MATH 539 Numerical Analysis (3-0)3 ECTS 8
Error analysis, direct and iterative methods for linear systems of equations, solution of nonlinear equations, and systems of nonlinear equations. Interpolation and approximation theory, numerical differentiation and integration.

MATH 540 Numerical Solution of Partial Differential Equations (3-0)3 ECTS 7
Finite difference schemes for parabolic, hyperbolic, elliptic equations. Order of the Accurancy of finite difference schemes. Stability of and convergence for difference schemes. Leapfrog, Lax-Wendroff, implicit, ADI methods, SOR, direct methods for partial differential equations

MATH 541 Ring Theory (3-0)3 ECTS 8
Ideals, Prime and Maximal Ideals, Nilradical and Jacobson Radical of a ring, The Socle of a ring, Simple and semisimple rings, Idempotents, Structure of semisimple rings: The Weddeburn-Artin Theorem, Local and semilocal rings, Semiperfect rings, Perfect rings, Semiregular rings, Regular rings, The ring of full linear transformations, Artinian and Noetherian rings, Quasi-Frobenius rings.

MATH 542 Algebraic Number Theory (3-0)3 ECTS 8
Euclidean, Noetherian, and Dedekind integral domains; integral elements; extensions of a fied; integral

bases for algebraic number fields; norm of ideals; factorization of primes in a number field; class group; Dirichlet's unit theorem; Diophantine equations.

MATH 543 Commutative Algebra I (3-0)3 ECTS 7
Commutative rings. Prime ideals and maximal ideals. Nilradical and Jacobson radical. Operation on ideals. Modules over commutative rings. Nakayama Lemma.Tensor product of modules.Restiriction and exlension of scalars. Exactness property of tensor product. Rings and modules of tractions. Local properties. Exlended and contracled ideals in rings of tractions.

MATH 544 Commutative Algebra II (3-0)3 ECTS 7
Primary decomposition. Integral dependence. Valuation rings. Chain conditions on modules. Commutative Noetherian rings. Primary decomposition in Noetherian rings. Commutative Artin rings. Discrete valuation rings. Dedekind domains. Fractional ideals. Dimension theory. Regular local rings.

MATH 546 Advanced Module Theory (3-0)3 ECTS 7
Category of modules. Generators and cogenerators.M-generated modules. Category σ(M). Generators in σ(M). M-injective modules. Self-injective modules. M-projective modules. Local rings. Finitely presented modules. Inverse limits. Finitely copresented modules. Finite uniform dimension. Complements and uniform dimension. Extending modules. Locally Noetherian extending modules. Locally Artinian modules.

MATH 547 Selected Topics in Rings and Modules (3-0)3 ECTS 7
Commutative domains and their modules; Decompositions of torsion and torsion-free modules.

MATH 548 Introduction to Algebraic Geometry (3-0)3 ECTS 7
Affine algebraic sets, Hilbert Basis Theorem, Hilbert's Nullstellensatz, affine varieties, local properties of plane curves.

MATH 549 Selected Topics in Algebraic Geometry (3-0)3 ECTS 7
Projective plane curves, Bezout's Theorem, Varieties, Morphisms, Resolution of singularities, Divisors.

MATH 554 Brownian Motion and Schrödinger's Equation (3-0)3 ECTS 7
Basic concepts. Killed Brownian motion. Schrodinger's operator. Stopped Keynman-Kac functional, conditional Brownian motion and conditional gauge. Green functions. Condition gauge and q-Green function.

MATH 562 Applied Functional Analysis (3-0)3 ECTS 7
The first part of the course briefly reviews some abstract results in Functional Analysis and operator theory as well as the function spaces related to the modern partial differential equations (PDEs) which appear in differential geometry, harmonic analysis, engineering, mechanics, and physics. The second most important part of the course teaches how the abstract results from Functional Analysis can be applied to solve PDEs.

MATH 563 Introduction to Finite Elements (3-0)3 ECTS 7
Variational formulation of elliptic boundary value problems. Galerkin-Ritz approximation. Finite element interpolation in Sobolev spaces. Error estimates. Computer implementation of finite element method(FEM). Stabilized FEMs.

MATH 564 Functional Analysis (3-0)3 ECTS 8
Metric spaces. Banach and Hilbert Spaces. Linear Operators on Normed Spaces, Bounded and Compact Operators. Spaces of Linear Operators and Convergence. Fundamental Theorems for Normed and Banach Spaces: Hanh-Banach Theorem, Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem. Linear Functionals on Hilbert Spaces and Riesz Representation

Theorem. Adjoint, Self-adjoint, Unitary and Normal Operators. Spectrum and Resolvent of an Operator. Spectral Properties of Bounded and Compact Operators. Unbounded Operators and Their Basic Properties.

MATH 565 Introduction to Spectral Theory (3-0)3 ECTS 7
Hilbert spaces. Spectral theory in finite dimensional spaces. Spectral properties of bounded and compact linear operators. Spectral theorem of bounded normal operators. Spectral representation of bounded self-adjoint operators. Unbounded linear operators and their adjoints. Closed operators. Spectral representation of unitary operators. Spectral representation of unbounded self-adjoint operators. Regular Sturm-Liouville operators. Linear operators in quantum mechanics.

MATH 566 Mathematical Foundations of Finite Element Method (3-0)3 ECTS 7
Theoretical foundations of finite element method for elliptic boundary value problems. Sobolev spaces. ınterpolation in Sobolev spaces. Variational formulation of elliptic boundary value problems. Basic error estimates. Applications to fluid dynamics. Practical aspects of the finite element method.

MATH 567 Mathematical Methods of Quantum Mechanics I (3-0)3 ECTS 7
Basic concepts. Complex vector space. Linear operators. The Schrodinger equation. Groups. Theoretical methods and symmetries. Dynamical symmetries and spectrum generating algebra.

MATH 568 Mathematical Methods of Quantum Mechanics II (3-0)3 ECTS 7
Fock space. The second quantization. Exactly solvable multi-particle problems. The coherent states. Fields quantization. Functional integral quantization.

MATH 569 Basic Quantum Computation and Quantum Information (3-0)3 ECTS 7
After providing the necessary background material in classical computation and quantum mechanics, the basic principles will be developed and the main results of quantum computation and information will be discussed.
Introduction to Classical Computation: The Turing machine. The circuit model of computation. Computational complexity. Energy and information. Reversible computation.
Introduction to Quantum Mechanics: The Stern-Gerlach experiment. Young’s double-split experiment. Linear vector spaces. The postulates of quantum mechanics. The EPR paradox and Bell’s inequalities.
Quantum Computation: 1. The qubit. The Bloch sphere. Measuring the state of a qubit. 2. The circuit model of quantum computation. 3. Single-qubit gates. Rotation of the Bloch sphere. 4. Controlled gates and entanglement generation. The Bell basis. 5. Universal quantum gates 6. Unitary errors 7. Function evaluation 8. The quantum adder. 9. Deutsch’s algorithm 10. Quantum search. 11. The quantum Fourier transform. 12. Quantum phase estimation 13. Period finding and Shor’s algorithm 14. Quantum computation of dynamical systems 15. Quantum simulation of the Schrodinger equation.
Quantum Communication: The no-cloning theorem. Faster-than-light transmission of information. Quantum teleportation.

MATH 570 Theory of Bounded Operators (3-0)3 ECTS 7
Introduction of compact operators, self adjoint operators, positive operators and their basic and spectral properties

MATH 571 Mathematical Methods of Classical Mechanics I (3-0)3 ECTS 7
Basic concepts of analytical mechanics. Generalized coordinates. Variational principles of mechanics. Hamilton’s principle of least action. Euler-Lagrange equations. Lagrangian for a system of particles. Conservation laws. Energy, momentum, angular momentum. Mechanical similarity and virial theorem. Systems with one and two degrees of freedom. Phase flow. Motion in a central field. Kepler’s problem. Elastic collisions. Rutherford’s formula. Small oscillations. Normal coordinates. A chain of coupled oscillators. Rigid body motion. The Euler angles. The Cayley-Klein parameters. Inertia tensor. The Euler top. The Lagrange top.

MATH 572 Mathematical Methods of Classical Mechanics II (3-0)3 ECTS 7

Hamiltonian mechanics. Legendre’s transformation. Hamilton’s equations. Hamilton’s function and energy. Cyclic coordinates. Routh’s function. Variational principle. The action as a function of coordinates. Maupertui’s and Fermat’s principles. Poisson brackets. Momentum space. Hamiltonian dynamics in rotating frame. Canonical transformations. Geometrical theory of the phase space. Symplectic structure. Infinitesimal canonical transformations. Conservation theorems and Poisson brackets. Hamilton’s mechanics in arbitrary variables. Hamilton-Jacobi equation. Principal and characteristic functions. Separation of variables. Central force problem. Action-angle variables.

MATH 573 Modern Geometry I (3-0)3 ECTS 8
Groups of transformations of Euclidean and pseudo-Euclidean spaces. The theory of curves. The theory of surfaces in three-dimensional space. The Riemannian metric. The second fundamental form. The Poincare model of Lobachevsky’s geometry. The complex geometry. Surfaces in complex space. The conformal form of the metric on a surface. Isothermal co-ordinates. Gaussian curvature in terms of conformal co-ordinates. Surfaces of constant curvature. The Fundamental Theorem of Surfaces. Gauss-Weingarten equations. Theorema Egregium of Gauss. Surfaces of constant negative curvature and the “Sine-Gordon” equation. Minimal surfaces. The Concept of a Manifold and the simplest Examples.

MATH 574 Modern Geometry II (3-0)3 ECTS 7
Tensors. Algebraic Theory and transformation rules. Skew-symmetrical tensors. Differential forms. Tensors in Riemannian and pseudo-Riemannian spaces. Vector fields and Lie algebras. The Lie derivative. The fundamental matrix Lie algebras. The exterior derivative and integration of differential forms. The general Stokes formula. Differential forms on complex spaces. The Kahlerian metrics. The curvature form. Covariant differentiation and the metric. Parallel transport of vector fields. Geodesics. The Riemann curvature tensor. The general curvature tensor. The symmetries of the curvature tensor. Examples of the curvature tensor in spaces of dimensions 2 and 3. The simplest concepts of the general theory of relativity.

MATH 575 Integral Equations (3-0)3 ECTS 7
Classification of integral equations. Integral equations solvable by integral transforms. Fredholm's theory of the linear integral equation of the second kind. Volterra's integral equation of the second kind. Volterra's integral equation and its solution by Lioville's iteration method. Study of linear integral equations by Schmidt's method.

MATH 576 Introduction to Soliton Theory (3-0)3 ECTS 7
IVP for Burgers’ equation and Cole-Hopf transformation. Shock solitons and their dynamics. Backlund transformation. General solution of the Liouville equation. The Sine-Gordon equation. Topological soliton. Bianchi permutability theorem and nonlinear superposition principle. Multi-soliton solutions. Collisions and bound states of solitons. From Riccati equation to the inverse scattering transform. Zero curvature and Lax representations. Zakharov-Shabat problem. Hirota direct method in soliton theory. KdV equation and the Schrodinger spectral problem. Elements of quantum scattering theory. The inverse problem. Gel’fand-Levitan-Marchenko equation and N-soliton solution. Analytic properties of the scattering amplitude. Integration of the KdV equation. Infinite hierarchy of integrals of motion. Current developments in soliton theory.

MATH 577 Supersymmetric Quantum Mechanics (3-0)3 ECTS 7
Review of Schrodinger equation. Factorization method. Construction of the hierarchy of Hamiltonians. Partner Hamiltonians. Examples- Harmonic oscillator and Morse potential. Supersymmetry and the radilal Problem. Isotropic oscillator. Breaking of SUSY in Quantum Mechanics. Supersymmetric WKB Approximation. Witten’s index. Examples: Electron in magnetic field.

MATH 578 Asymptotic Analysis (3-0)3 ECTS 7
Asymptotic expansions. Integration by parts. The method of stationary phase. The method of Laplace. Watson’s lemma. The method of steepest descents. The saddle-point method. Airy’s integral. Singularities of differential equations.

MATH 579 Applied Complex Analysis (3-0)3 ECTS 7
Analytic functions. Infinite Products. Cauchy Integrals. Conformal transformations. Mobius transformations. Christofel-Schwarz conformal transformations. Conformal metric and geometry. Boundary value problems in Electrostatics and hydrodynamics.

MATH 581 Topology I (3-0)3 ECTS 7
Topological spaces. Subspaces. Bases of open sets. Subbases. Neighborhood systems. Separation axioms. Interior. Closure. Derived Set. Fronitier of subsets of a topological space. Continuous functions. Compact spaces. Tychonoff theorem. Compactness in $\mathbf{R}^n$. Connected spaces. Path connected spaces.

MATH 582 Topology II (3-0)3 ECTS 7
Locally compact topological spaces. Paracompactness. Compactifications. Metrizable spaces. Urysohn's metrization theorem. Complete spaces. Completion of metric spaces. Locally connected spaces. The concept of dimension.
Pre. MATH 581 Topology I

MATH 584 Partial Differential Equations II (3-0)3 ECTS 8
Sobolev spaces, Elliptic equations, Linear evolution equations.

MATH 585 Symmetries and Groups (3-0)3 ECTS 7
Basic group theory and representations. Symmetry groups in physics. Groups and differential equations.

MATH 586 Hilbert spaces and Quantum theory (3-0)3 ECTS 7
The basic concepts of classical and quantum mechanics. Quantum field theory. Classical topology and Quantum states. Supersymmetric Quantum Theory and the Index Theorem

MATH 587 Mathematics of Public Key Cryptography (3-0)3 ECTS 7
An in depth study of public key cryptography and number theoretic problems related to the efficient and secure use of public key cryptographic schemes.

MATH 588 Mathematical Control Theory for Infinite Dimensional Systems (3-0)3 ECTS 7
Exact controllability, stabilization, multiplier method.

MATH 589 Advanced Combinatorics (3-0)3 ECTS 7
Topics of instructor's choice in discrete mathematics, algorithms, graph theory, combinatorics and their applications.

MATH 590 Selected Topics in Applied Mathematics (3-0)3 ECTS 7
Selected topics in mathematical problems arising from various applied fields such as mechanics, economics, etc.
Pre. Consent of instructor.

MATH 595 Graduation Project (0-2)NC ECTS 5
Students must complete a project in the basic areas of mathematics and then must submit a written report to the lecturer.

MATH 596 Graduate Seminar (0-2)NC ECTS 9
Oral presentations on topics dealing with current research and technical literature. Includes presentation of latest research results by quest lecturers, staff and advanced students.

MATH 597 Comprehensive Studies (0-2)NC ECTS 9

Students must complete four projects in the basic areas of mathematics and then must pass a written exam in each project. Four projects can be taken in the following topics: Algebra, Real Analysis, Complex Analysis, Functional Analysis, Ordinary Differential Equations, Partial Differential Equations, Geometry, Topology, Numerical Analysis.

MATH 598 Graduate Seminar in PhD (0-2) NC AKTS:9

The course is composed of the research, analyze, data collecting and reporting the results of the Ph.D student relating to the specific subject the student aims to study.

MATH 599 Scientific Research Techniques and Publication Ethics (0-2) NC AKTS:8

Scientific methods, testing scientific hypotheses, scientific writing and publishing. Basic principles, integrity, authorship, responsible publication, references and citations. Sloppy research, scientific fraud, plagiarism, fabrication, duplication, ghost authorship.

MATH 500 M.S. Thesis (0-1)NC ECTS 26

Program of research leading to M.S. degree arranged between student and a faculty member. Students register to this course in all semesters starting from the begining of their second semester while the research program or write-up of thesis is in progress.

MATH 600 Ph.D. Thesis (0-1)NC ECTS 26

Program of research leading to Ph.D. Degree arranged between student and a faculty member. Students register to this course in all semesters starting from the beginning of their second semester while the research programme or write-up of thesis is in progress.

MATH 8XX Special Studies (8-0)NC ECTS 4

Graduate students supervised by the same faculty member study advanced topics under the guidance of their advisor.